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A NUMERICAL STUDY OF THE ELASTO-HYDRODYNAMIC LUBRICATION PROBLEM

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This investigation thoroughly studies the Elastohydrodynamic Lubrication (EHL) problem, focusing on the complex behavior of lubricants at high pressures and varying sliding velocities. The normalized Reynolds equation is solved using the finite difference method (FDM) in this study, which focuses on the impacts at higher beta values, specifically around , a novel regime where is a dimensionless parameter. Using a shifted sigmoid function to modulate sliding velocities and providing more detailed knowledge of the lubrication dynamics is a crucial component of this research. Furthermore, in order to properly model the deformation of lubricated surfaces under load, the research investigates the link between the Reynolds equation and the elastic deformation equation. The results of this study provide a significant understanding of the intricate relationship between fluid dynamics and elastic deformation in EHL, which is beneficial in optimizing lubrication performance in engineering applications. In addition to improving the theoretical foundation of EHL, this study has application in the design and maintenance of mechanical systems where effective lubrication is essential.
Keywords: *Elasto-Hydrodynamic Lubrication, Reynolds' Equation, Elastic Deformation, Tribology, Finite Differences.*

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Introduction

When two solid surfaces are separated by a thin lubricating film and subjected to high pressures, a phenomenon known as Elastohydrodynamic lubrication (EHL) occurs. Unlike boundary lubrication, where surfaces come into direct contact, EHL relies on the elastic deformation of both the contact surfaces and the lubricant film under pressure. This interplay enables effective separation, minimizes friction, and significantly reduces wear [11].

The high pressures in EHL cause the lubricant film to deform elastically, leading to an increase in the film thickness. This deformation generates a hydrodynamic effect that supports the load and prevents direct surface contact, as shown in Figure 1.1. Consequently, EHL plays a critical role in reducing wear, lowering friction, and

Extending the operational life of mechanical systems.

EHL phenomena are of paramount importance in the field of tribology, which is the study of friction, wear, and lubrication of interacting surfaces in relative motion. A comprehensive understanding of EHL is essential for optimizing the design and performance of lubricated systems, such as bearings, gears, and camshafts[2].

Extensive research on EHL has been conducted using experimental, theoretical, and numerical approaches. Experimental studies involve specialized tools to measure lubricant film thickness, pressure distribution, and frictional forces under controlled conditions. Theoretical investigations employ mathematical models, such as the Reynolds equation, to describe the behavior of lubricating films and calculate parameters like pressure distribution and load capacity [7].

Numerical methods, including finite difference, finite element, and boundary element approaches, have further advanced the study of EHL. These methods allow for detailed predictions of film thickness profiles, pressure distributions, and frictional forces, enabling engineers to optimize system designs and lubricant selection.[1]

1.1 Background and Historical Context

The foundations of modern lubrication theory can be traced back to the late 19th century with the work of Osborne Reynolds, whose groundbreaking research in fluid dynamics laid the groundwork for understanding fluid motion in various contexts. Engineers at the time faced significant challenges in predicting and controlling fluid flow, particularly in pipes and channels.

Reynolds developed the now-famous Reynolds equation, which describes the behavior of fluid films under hydrodynamic conditions. His experiments introduced dimensionless numbers, such as the Reynolds number, to characterize flow regimes. These contributions provided a mathematical framework for analyzing the transition from laminar to turbulent flow, influencing subsequent advancements in lubrication theory [4].

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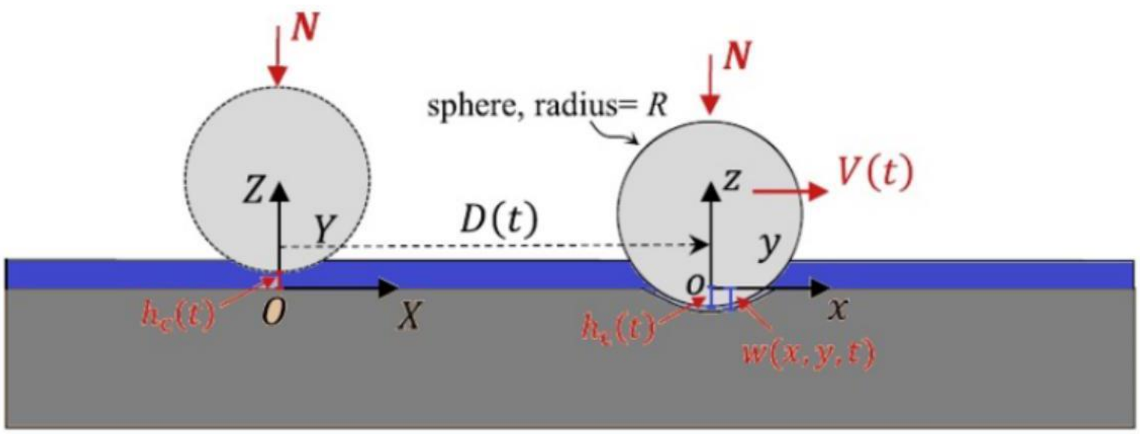

Figure 1: A Model Problem showing a sliding rigid sphere on an elastic substrate [12]

1.2 Hydrodynamic and Elastohydrodynamic Lubrication

Lubrication can be broadly classified into hydrodynamic lubrication and Elastohydrodynamic lubrication (EHL), depending on the nature of the contact and the pressures involved.

In hydrodynamic lubrication, typical of conformal contacts like journal and thrust bearings, the pressure generated is insufficient to cause significant elastic deformation of the contacting surfaces. In contrast, EHL occurs in concentrated contacts, such as those in gears, roller bearings, and cam mechanisms, where the load is distributed over a small area (often just a few square millimeters).

Under EHL conditions, contact pressures can reach up to 4 GPa, causing substantial elastic deformation of the mating surfaces. Simultaneously, the viscosity of the lubricant increases dramatically - by several orders of magnitude - compared to atmospheric conditions, altering the flow mechanics significantly [10].

1.3 Lubrication Regimes

EHL operates within distinct lubrication regimes depending on the load, speed, and lubricant film thickness. These regimes are:

- Boundary lubrication: Occurs under extreme conditions where the lubricant film is too thin to prevent direct contact, leading to significant asperity interactions.
- Mixed lubrication: Involves partial surface contact, with both asperity interactions and hydrodynamic effects contributing to the load-carrying capacity.
- Full-film lubrication: The lubricant film is sufficiently thick to completely separate the contacting surfaces, minimizing wear and reducing the coefficient of friction [3].

Systems operating in the EHL regime aim to maintain full-film lubrication to maximize efficiency and component lifespan. However, real-world conditions - such as high loads or low speeds - can shift the system into mixed or boundary regimes, necessitating careful consideration of lubricant properties and system design.

1.4 Research Gaps

Unquestionably, the advancement of computer technologies and the development of numerical analysis based on mathematical methodologies have led to a great deal of fundamental understanding of lubricating phenomena. More specifically, a basic comprehension of pressure accumulation and film generation in an Elastohydrodynamically lubricated contact. Such numerical models have contributed substantially to engineering discoveries. It is computationally expensive to apply such techniques in multibody dynamic (MBD) system simulations. This increases the possibility of a delayed convergence and an unwanted interruption, especially if the entire ICE engine is treated as an MBD system. The line and elliptical/point contacts are more frequently the subject of numerical simulations than the real shape of the contacting.

Bodies. The ellipse becomes truncated in a variety of scenarios when the contact expands to the roller's edges. Highly developed models based on the Reynolds and energy equations as the fundamental equations for the film development, pressure build-up, and temperature distribution inside the lubricated combination can be used to numerically predict Elastohydrodynamic friction [6].

2 Governing Equations

When one of the contacting bodies is elastically soft, several significant issues with Elastohydrodynamic lubrication develop. As an illustration, consider the case of squeezing a thin liquid film between a rigid sphere or cylinder and a soft elastic substrate. In this scenario, the sphere may not slide and be merely subjected to normal loads (stationary contact), or it may be sheared simultaneously and slide steadily or unsteadily. When soft substances are subjected to heavy loads and thin lubricant layers, these issues are notoriously challenging to resolve. Because of this, indentation and sliding issues have typically been studied independently, and transitory sliding issues have limited answers. The Reynolds equation in cylindrical coordinates is solved using a novel finite difference scheme for the model problem by coupling hydrodynamic pressure with substrate surface displacement. The results are studied in three different phases. Equations in the cylindrical coordinates are given by [11],

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{12\eta}r\frac{\partial p}{\partial r}\cdot d^3\right)+\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{12\eta}\frac{1}{r}\frac{\partial p}{\partial\theta}\cdot d^3\right)=-\frac{V(t)}{2}\left(\cos\theta\frac{\partial d}{\partial r}-\frac{\sin\theta}{r}\frac{\partial d}{\partial\theta}\right)+\frac{\partial d}{\partial t}. \tag{2.1}$$

$$w(r,\theta)=-\frac{1}{4\pi G}\int_0^{2\pi}\int_0^{\infty}\frac{p(r',\theta')r'dr'd\theta'}{\sqrt{(r\cos\theta-r'\cos\theta')^2+(r\sin\theta-r'\sin\theta')^2}} \tag{2.2}$$

$$d=h-w. \tag{2.3}$$

Since the fluid pressure fluctuates slowly in some directions, using a cylindrical coordinate system minimizes the number of elements in the finite difference mesh. We present the ensuing normalisation.

$$\bar{r}=\frac{r}{\sqrt{Rh_\infty}},\bar{t}=\frac{V_{ss}t}{2\sqrt{Rh_\infty}},\bar{w}=\frac{w}{h_\infty},\bar{h}=\frac{h}{h_\infty},\bar{d}=\frac{d}{h_\infty},\bar{p}=\frac{p}{4\pi G\sqrt{h_\infty/R}}. \tag{2.4}$$

3 Hertzian Theory

Hertzian theory, sometimes referred to as Hertzian contact theory or Hertz contact theory, is a key idea that is used to examine and comprehend how solid substances behave when they come into contact under extreme pressure [14]. The Hertzian theory offers a basis for investigating the contact between two surfaces and the ensuing deformation and stresses in the context of Elastohydrodynamic lubrication (EHL)[8].

Heinrich Hertz created the Hertzian theory in the late 19th century to explain the interaction of elastic bodies. It is predicated on the supposition that the contacting surfaces will not be rough and that the deformations of the bodies will be elastic, meaning they won't be permanent.

When analyzing the contact between rolling or sliding surfaces, as in rolling element bearings or gear systems, EHL employs the Hertzian theory. Due to the imposed load, the surfaces deform elastically as they come into contact, forming a contact area and producing stresses in the materials.

3.1 Limitations of Hertzian Theory

The Hertzian theory is limited, even though it offers a useful view of contact and deformation. It assumes that elastic deformation will follow a straight path and ignores the effects of surface wear, uneven loading, and lubricant presence. Therefore, more theories and models, including Reynolds' equation and numerical approaches, are helpful to account for the complex interactions between the surfaces and the lubricant coating to provide a more precise characterization of EHL.

To conclude, this theory gives an important idea in the study of the interaction between elastic materials. It offers a footing for telling the initial contact and deformation of rolling or sliding surfaces in EHL. This theory provides computation of the maximum contact stresses, contact pressures, and contact area geometry. Although Hertzian theory has its limitations, it is nevertheless a useful starting point for understanding how contacting bodies behave in EHL and serves as the basis for more difficult models and numerical methods[9] [15].

4 Numerical Solution Using FDM

Numerous numerical techniques can be used to solve Reynolds' equation, a partial differential equation that describes fluid flow in thin-layer lubrication regimes. By using these techniques, engineers and researchers can analyze the pressure distribution, load-carrying capacity, and other flow properties in lubricated systems and derive approximations of the solutions to Reynolds' equation. This section will provide a thorough examination of the numerical techniques, such as the finite difference, finite element, and boundary element approaches, that are frequently used to solve Reynolds' equation.

A common approach is the finite difference method for solving Reynolds' equation numerically. In this approach, Reynolds' equation is discretized on a grid, and the spatial derivatives are estimated using finite difference approximations. To create an algebraic system of equations, the spatial coordinates' derivatives are swapped out for finite difference formulas like the centered difference and the backward difference. Then, by employing techniques like the Gauss-Seidel or Successive Over-Relaxation (SOR) method, these equations can be solved iteratively. The size of the grid and the order of the finite difference algorithm utilized both affect how accurate the solution is.

Explicit Scheme: In this scheme, forward differences in time and space are used

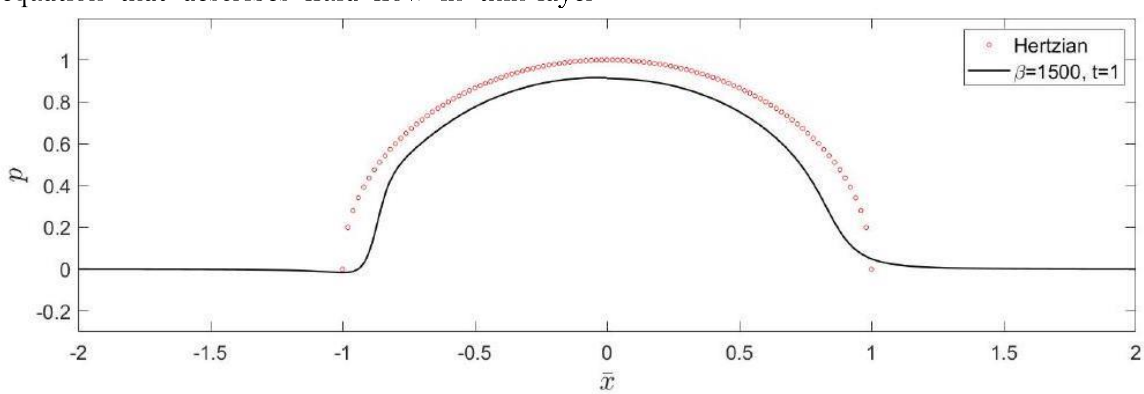

Figure 2: The results are obtained at Beta = 1500, and it can be seen within the hertzian limit

To approximate the derivatives of the Reynolds equation. Despite being computationally straightforward, the explicit nature may necessitate very short time steps to guarantee stability. Since the explicit scheme is conditionally stable, care must be taken while selecting the time step to avoid instability.

Implicit Scheme: The derivatives are

approximated using backward differences in time and space. The resulting system of equations is typically solved iteratively using strategies like sequential over-relaxation or Gauss-Seidel. Larger time steps and improved stability are possible with the implicit system, but the cost is an increase in computational complexity.

The benchmark model analyzes the transient pressure distribution and film height as shown in Fig. 2 and Fig. 3, in a squeeze-film bearing for lubrication in a nonconformal conjunction of a solid sphere and an elastic wall where a lubricant film separates them. Lubrication is the practice of pouring a liquid substance into the middle of mechanical parts to reduce the frictional effect that arises during their contact. Elastohydrodynamic contact is the contact of mechanical parts from the Elastohydrodynamic perspective. It is an interaction between a lubricant and elastic bodies; there is the consideration of pressure developed in the lubricant and the mechanical stresses in the neighborhood of the contact center. Most of such problems are solved numerically. The problem is modelled by solving Reynolds' equation, and solid mechanics as a coupled problem is solved simultaneously.

The novel method proposed by Haibin Wu and Anand Jagota [13] works for large beta values such as $\beta = 1200$ and $\beta = 1500$. But when β goes beyond 1500, that is normal load is increased, then the hertzian limit is broken, which is shown in Figure 3. Various β values have been tested.

This problem is resolved by a little modification in the velocity function of the sliding sphere. Initially sliding velocity was given by V tanht, which doesn't work for beta values higher than 1500, but when we modify this function by $V = \frac{1}{1+e^{10-t}}$, the solution remains in the hertzian limit, which is shown in Fig. 5. This shows that by controlling the sliding velocity of sphere one may get appropriate results for higher loads as large beta value implies large loads.

To discretize this equation, the computational domain is divided into a grid as shown in Fig 4, and finite difference approximations are used to replace the partial derivatives with algebraic expressions.

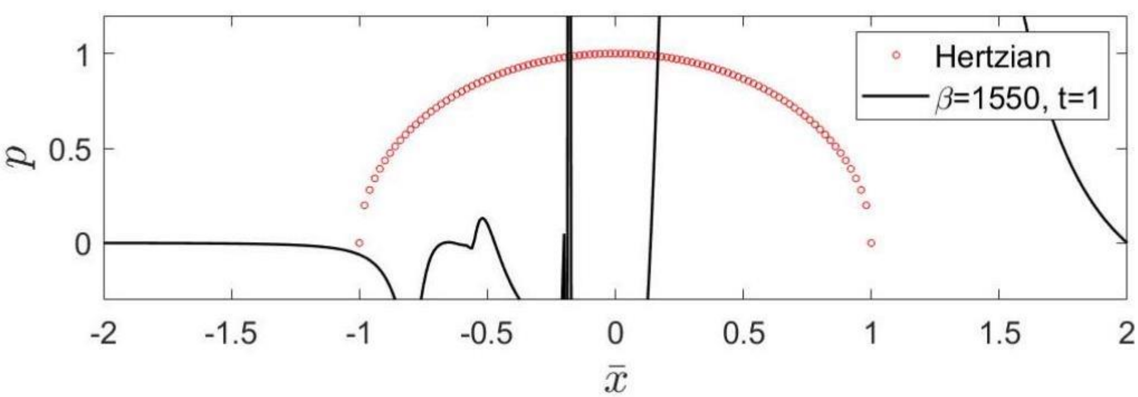

Figure 3: The result is obtained at Beta = 1550 and can be seen that the problem diverges by breaking Hertzian limit.

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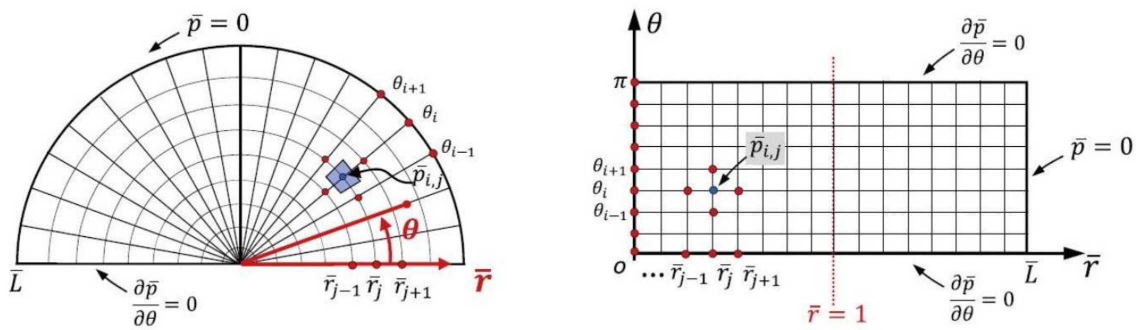


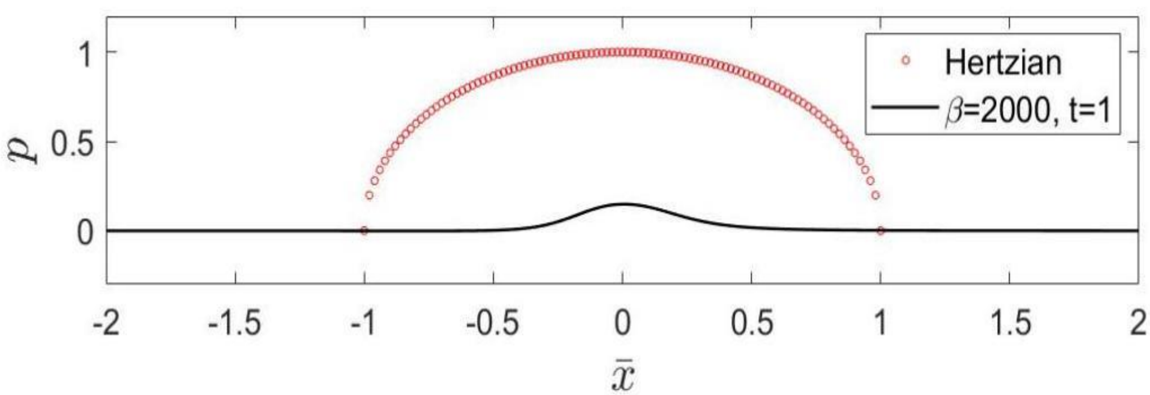
Figure 4: On the right hand side is the zoomed part of the polar grid Grid [11]

$$\frac{\partial}{\partial X}\left(\bar{h}^3 \frac{\partial \bar{p}}{\partial X}\right) \approx \frac{\bar{h}^3_{i+1/2,j} \frac{\bar{p}_{i+1,j} - \bar{p}_{i,j}}{\Delta X} - \bar{h}^3_{i-1/2,j} \frac{\bar{p}_{i,j} - \bar{p}_{i-1,j}}{\Delta X}}{\Delta X}. \tag{4.1}$$

$$\frac{\partial}{\partial Z}\left(\bar{h}^3 \frac{\partial \bar{p}}{\partial Z}\right) \approx \frac{\bar{h}^3_{i,j+1/2} \frac{\bar{p}_{i,j+1} - \bar{p}_{i,j}}{\Delta Z} - \bar{h}^3_{i,j-1/2} \frac{\bar{p}_{i,j} - \bar{p}_{i,j-1}}{\Delta Z}}{\Delta Z}. \tag{4.2}$$

$$\frac{\partial \bar{h}}{\partial X} \approx \frac{\bar{h}_{i+1/2,j} - \bar{h}_{i-1/2,j}}{\Delta X}. \tag{4.3}$$

An important part of lubrication theory, the Reynolds equation describes how pressure is distributed in lubricant films that parts the surfaces moving relative to one another. Real-world applications frequently deal in complex shapes, even though closed form solutions exist for simple geometries. In this sort of situations, Finite



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VOLUME . 4 ISSUE . 1 (2025)

Figure 5: Using shifted sigmoid function obtained results remain in hertzian limit at Beta $= 2000$

Element Analysis (FEA) proves to be a good tool for solving the Reynolds equation numerically. Simple geometries such as infinitely long journal bearings or infinitely wide sliders are the only analytical solutions available for the Reynolds equation. FEA becomes crucial in situations involving:

Changeable attributes: Numerical solutions capable of handling such nonlinearities are needed because lubricant viscosity can change as a function of temperature and pressure. Cavitation: The fluid film may occasionally burst, introducing areas of zero pressure known as cavitation. Cavitation models can be incorporated into FEA for more precise analysis [5].

5 Conclusion

Based on the results obtained, it is evident that using a large beta value in the Elasto-Hydrodynamic Lubrication (EHL) model with the sliding velocity function $V = \tanh(t)$ led to the Hertzian limit being exceeded when beta was set to 1500 . However, modifying the function to a shifted sigmoid form $V = \frac{1}{1+e^{10-t}}$ produced more appropriate results even at larger beta values, such as beta $= 2000$. This adjustment suggests that the shifted sigmoid function effectively managed the lubrication conditions under higher pressure scenarios, highlighting its suitability in maintaining model accuracy beyond traditional limits.

Furthermore, the numerical resolution of the Reynolds equation concerning the soft lubrication issue has yielded significant understanding of the behavior of lubricated systems in diverse operating scenarios. The complex relationships between pressure gradients, film thickness, and load distribution-all of which are essential for maximizing lubrication performance - are made clear by the numerical solutions. These solutions improve our knowledge of how material qualities affect lubrication efficacy by taking into account the soft, malleable nature of the relevant surfaces. This information will eventually help build more dependable and efficient lubrication systems.

On the other hand the transient Elastohydrodynamic lubrication squeezing problem reveals the intricate dynamics between hydrodynamic pressure, film thickness, and the applied load. The study demonstrates that as the load is applied, the lubricant film undergoes rapid compression, leading to significant variations in pressure and film thickness over time. The agreement between numerical and analytical results validates the accuracy and reliability of the computational model employed.

Understanding these dynamics is critical for advancing the design and optimization of mechanical systems that rely on lubrication for efficient operation. This study highlights the importance of coupling the deformation of elastic surfaces with lubricant flow to predict transient behaviors accurately. Such insights are instrumental in preventing surface damage, reducing friction, and extending the lifespan of components in applications ranging from automotive to aerospace industries.

The implications of these findings are profound, paving the way for more robust and efficient EHL systems. Future research could focus on incorporating more complex geometries, material properties, and non-Newtonian lubricant behaviors to further enhance the predictive capabilities of EHL models. By bridging the gap between theoretical modeling and practical application, this work contributes to the ongoing development of innovative solutions for modern engineering challenges.

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